REPORT CRPL-9-7 ISSUED OCTOBER 1, 1948

National Bureau of Standards
Library, N. W. Bldg.
NOV 1 6 1949

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
CENTRAL RADIO PROPAGATION LABORATORY
WASHINGTON, D. C.

MICROWAVE FREQUENCY MEASUREMENTS AND STANDARDS

BY B. R. HUSTEN AND HAROLD LYONS

Reprinted from
TRANSACTIONS of the American
Institute of Electrical Engineers
Volume 67, page 436, 1948

Microwave Frequency Measurements and Standards

BENJAMIN F. HUSTEN
NONMEMBER AIEE

HAROLD LYONS
MEMBER AIEE

IN THIS PAPER a survey is given of the problem of microwave frequency measurement and standardization from the point of view of the method adopted at the National Bureau of Standards. The bureau's microwave frequency standard, methods of measurement, and accuracy of calibrations are discussed. A brief preview of possible future developments in this field is also given to indicate some of the lines of attack being considered for the extension and diversification of microwave frequency measurement methods and the application of these techniques to related problems.

National Bureau of Standards Microwave Frequency Standard

The National Bureau of Standards microwave frequency standard was designed to provide continuous frequency coverage from approximately 300 megacycles per second up to at least 30,000 megacycles per second. There is also provision for the generation of spot frequencies of highest precision at approximately 1 per cent intervals over the same band of frequencies. The completed unit was designed to be comparatively easy to operate and maintain, to be as free as possible of all spurious frequencies, and to provide sufficient power over the entire range so it would be possible to calibrate most microwave frequency meters without resorting to external voltage sources. The establishment of this microwave frequency standard was initiated by a request of the Joint Communications Board of the United States Joint Chiefs of Staff in 1944. The unit was put into operation in June 1945 with the help of the Massachusetts Institute of Technology Radiation Laboratory.

In order to keep the power requirement to as low a value as possible, sensitive spectrum analyzers or superheterodyne receivers with panoramic adapters are used as indicating equipment. The actual power available up to at least 26,000 megacycles per second is more than sufficient to perform most calibrations even though 10 decibel isolating pads are used on each side of the meter being calibrated.

The complete microwave frequency calibration equipment except the 100-kc primary standard is contained in one room which is well shielded and provided with temperature and humidity control equipment.

The standard microwave frequencies are obtained from the National primary standard of frequency at 100 kc through a system of frequency multiplication, frequency conversion, and harmonic selection. The National primary standard consists of nine quartz-crystal oscillators which are compared automatically with each other and with Naval Observatory time. The absolute value of the frequency of any one of these oscillators is known at all times to within one part in 100 million.

In Figure 1 is shown a picture of the complete microwave frequency generator. The center rack contains the multipliers, decade generator, frequency converters, and variable frequency generators necessary to obtain and multiply the desired frequency to 30 megacycles per second. The top units of the rack on the left are, respectively, the high-frequency multiplier stages and high-frequency filters. Immediately below these units is the spectrum analyzer with interchangeable microwave plumbing. This equipment is set up for the calibration of a 9,000-megacycle frequency meter. The equipment in the right-hand rack is used to determine precisely the frequency of the variable low-frequency oscillators when they are being used.

Additional equipment not shown in the picture but used during some calibration includes superheterodyne receivers and panoramic adapters which cover the range from 300 to 3,000 megacycles per second and various voltage generators which cover a considerable part of the spectrum between 300 and 30,000 megacycles per second.

Continuous Coverage System

The block diagram in Figure 2 shows the microwave standard set up for continuous coverage. This 100-kc output from the primary standard, after passing through a distribution amplifier, is multiplied to 7,500 kc. This frequency then is added to the output of one of two precision variable oscillators to give an output which is variable from 9.5 to 10.3 megacycles per second. Although, for simplicity, it is not shown in the block diagram, it is possible to add a fixed frequency to the output of either of these oscillators through the equipment used for spot frequency generation so as to obtain a frequency range from 2.0 to 3.0 megacycles per second, which when added to the 7.5 megacycle frequency results in an output variable from 9.5 to 10.5 megacycles per second. In order to reduce the level of undesired modulation products, particular attention was given to the design and construction of the frequency converter where the output of the variable oscillator is combined with the 7.5-megacycle frequency obtained from the primary standard.

The output of the converter is passed through four consecutive frequency triplers with outputs available in the vicinity

Paper 48-49, recommended by the AIEE instruments and measurements committee and approved by the AIEE technical program committee for presentation at the AIEE winter general meeting, Pittsburgh, Pa., January 26–30, 1948. Manuscript submitted November 12, 1947; made available for printing December 15, 1947.

BENJAMIN F. HUSTEN and HAROLD LYONS are both with the central radio propagation laboratory, National Bureau of Standards, Washington, D. C.

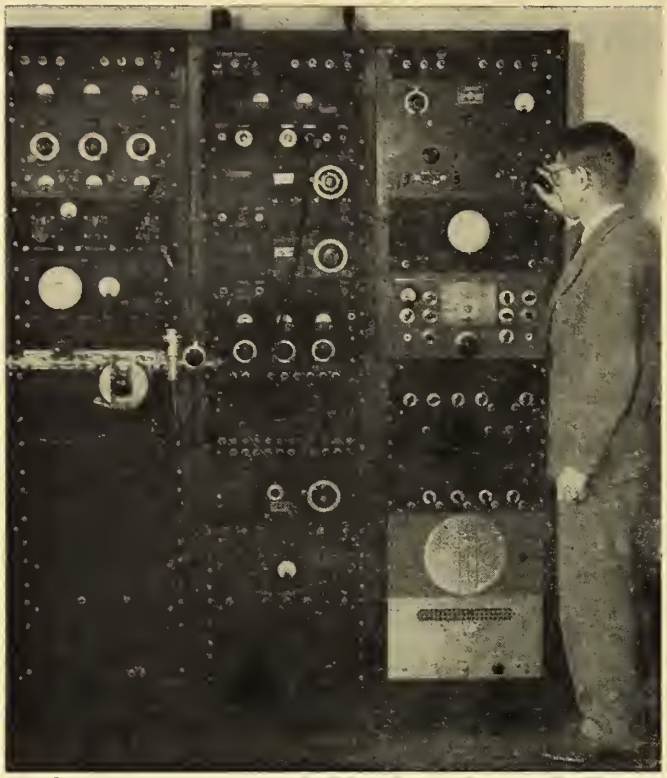


Figure 1. Microwave frequency standard

of 30, 90, 270, and 810 megacycles. Each of these stages has a tuning range of slightly greater than 10 per cent so as to accommodate only the range of variable frequency, making it impossible to tune to any except the proper harmonic. The output from any one of these four tripler stages can be impressed across a nonlinear element such as a crystal rectifier which will generate higher-order harmonics. This part of the equipment will be discussed in a following section. The frequency stability of any one of the harmonics will be directly dependent upon the output of the 10-megacycle frequency converter which, in turn, is about 75 per cent dependent upon the primary standard and only 25 per cent dependent upon the precision oscillators. These oscillators are equipped with micrometer dials which give a change in frequency of about five parts in a million per dial division. The instantaneous frequency of the oscillators can be measured to within one cycle by means of the equipment on the right of the block diagram. This equipment consists of a frequency divider, harmonic generators, a receiver, a comparison oscilloscope, and an audio-frequency interpolation oscillator.

Harmonics controlled directly by the primary standard are produced at intervals of 10 kc in the receiver. The unknown frequency and one of these harmonics beat together to produce a beat note which can be measured to within a cycle by means of the interpolation oscillator and the comparison oscilloscope. The interpolation oscillator can, in turn, be calibrated directly against the primary standard.

When the frequency of the variable oscillator is known to within one cycle, the 10-megacycle output and the resulting microwave harmonic are known to within one part in 10 million.

Spot Frequency System

At times a calibration is not desired at any particular frequency but at several points within a range of frequencies. In order to easily obtain such a series of precisely known frequencies the spot frequency generating equipment is used. Figure 3 shows a block diagram of the microwave standard set up for such use. One very apparent simplification is obvious from a comparison with the general coverage system as shown in Figure 2. No auxiliary frequency measuring equipment is needed since the frequency is at all times dependent directly upon the primary standard. The precision oscillators now are replaced by a system of multipliers and converters, called the decade generator, which provides an output adjustable in 100-kc steps from 2.0 to 3.0 megacycles per second. This is accomplished by adding to a 2.0-megacycle frequency the output of any one of a series of frequency multipliers which provide outputs every 100 kc from 100 to 1,000 kc. This frequency then is added to the 7.5-megacycle output and passed through the frequency multipliers as in the previous case. Any one of the outputs is now adjustable over an approximately 10 per cent range at intervals of 1 per cent. Since these frequencies are derived directly from the primary standard, they are known to within one part in 100 million.

Fine Tuning System

An additional arrangement of the equipment is possible, namely, the fine tuning arrangement. This consists of a precision oscillator variable between 500 and 600 kc which can be mixed with the outputs of the decade frequency generator and multiplier to give continuous coverage from 2.0 to 3.0 megacycles per second. The stability of the output from the 10-megacycle converter stage with this system is 95 per cent dependent upon the primary standard and about 5 per cent dependent upon the oscillator. The advantage of this system over the previous continuous coverage system is that each dial division of the oscillator changes the frequency about one part per million as compared to the change of five parts per million of the other oscillators. The frequency of the oscillator is determined by the interpolation equipment as in the previous case and the accuracy of the final frequency is within one part in 10 million.

Generation of Higher-Order Harmonics

Although the output from the microwave frequency standard is tunable over a relatively narrow frequency range, continuous frequency coverage can be obtained through the overlapping of harmonics. With the 10 per cent band width being used, harmonics of the output above the tenth overlap and provide continuous coverage. The 30-megacycle output provides continuous coverage from 285 megacycles, the 90-megacycle output from 855 megacycles, the 270-megacycle output from 2,565 megacycles, and the 810-megacycle output from 7,695 megacycles. The output of any one stage must be used up to the 30th harmonic before the harmonics of the succeeding stage begin overlapping. The power level of the 30th harmonic of any one of the stages is sufficient for frequency calibration purposes.

In order to generate these high-order harmonics the output from any one of the four frequency multiplier stages is impressed across a crystal diode mounted in a coaxial line or a section of wave guide, whichever the case may be. Figure 4

shows one of these crystals mounted in a section of 1-inch-by-1/2-inch wave guide intended to operate in the range from 8,500 to 10,000 megacycles. A typical crystal is shown in Figure 5 and a schematic of the crystal is shown in Figure 6. The crystal is mounted in the wave guide with appropriate tuners so it not only generates the harmonics, but also serves as an antenna for the efficient radiation of the microwave energy down the guide.

The power which these crystals can handle without overloading or burning out is limited. Some of the crystals designed to operate in the vicinity of 1,000 megacycles can handle several watts of power without being damaged, but most of the higher frequency crystals can handle only a few tenths of a watt.

Measurements made on the strength of the harmonics available from a *1N23* crystal diode indicate that with a power input somewhat less than 0.4 watt from the 810-megacycle stage the power level of the 11th harmonic at 8,910 megacycles is about 80 microwatts. The power available at the 30th harmonic of the same stage from a *1N26* crystal diode at 24,300 megacycle is about 0.05 microwatt. Since the sensitivity of the receiver is about 10^{-6} microwatt, there is sufficient power to perform most calibrations.

When a wave guide is used to propagate these higher-order harmonics, no difficulties are ever encountered with lower harmonics or with the fundamental itself since the wave guide transmission line is an excellent high-pass filter, passing only those frequencies which are above the cut-off frequency. When coaxial line is used for the transmission of the energy, it is sometimes necessary to employ tuned stubs or filters in the line to eliminate the stronger lower-order harmonics.

Receiving Equipment

Since superheterodyne receivers used with panoramic adapters are well known, no discussion will be given of this equipment which is used below 2,700 megacycles. The spectrum analyzer and its use, however, may warrant a slight description. The use of and precautions needed with this receiver will apply in general to the panoramic adapter combination.

The spectrum analyzer is essentially a superheterodyne receiver with a frequency-modulated local oscillator. A block diagram of a typical spectrum analyzer receiver is shown in Figure 7. The local oscillator, a klystron tube, is frequency-modulated by means of a saw-tooth wave. This same saw-tooth wave is impressed upon the horizontal plates of a cathode-ray tube. The video output of the receiver is impressed on the vertical plates of this tube. Whenever the difference between the input frequency and the frequency of the local oscillator is equal to the intermediate frequency of the receiver, a pip will appear on the screen of the cathode-ray tube. The height of this pip will be a function of the input power and the position on the screen will be a function of the input frequency and the center frequency of the local oscillator. There will be two such pips for each frequency although they may not be on the screen simultaneously. These pips correspond to instantaneous local oscillator frequencies above and below the input frequency. When the spectrum analyzer is used to compare two frequencies, it is possible for the two to appear to coincide when they are separated by twice the intermediate frequency. When the two frequencies are in proper coincidence the two pips above the local oscillator frequency will coincide as will the two below the local oscillator. Figure 8 shows a close-up view of the spectrum analyzer with one of the interchangeable wave guide assemblies used to cover the various frequency ranges corresponding to each wave guide size.

Measurement of Microwave Frequencies

In order to expedite the setting of the microwave frequency standard, tables to the frequencies of the various harmonics versus the variable frequency input into the 10-megacycle converter have been

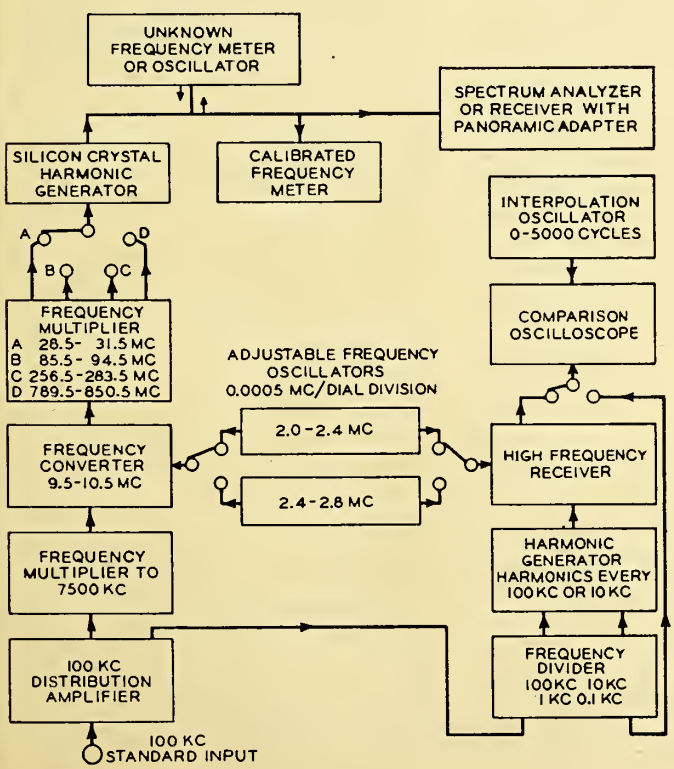


Figure 2. Microwave frequency standard continuous coverage with adjustable oscillators

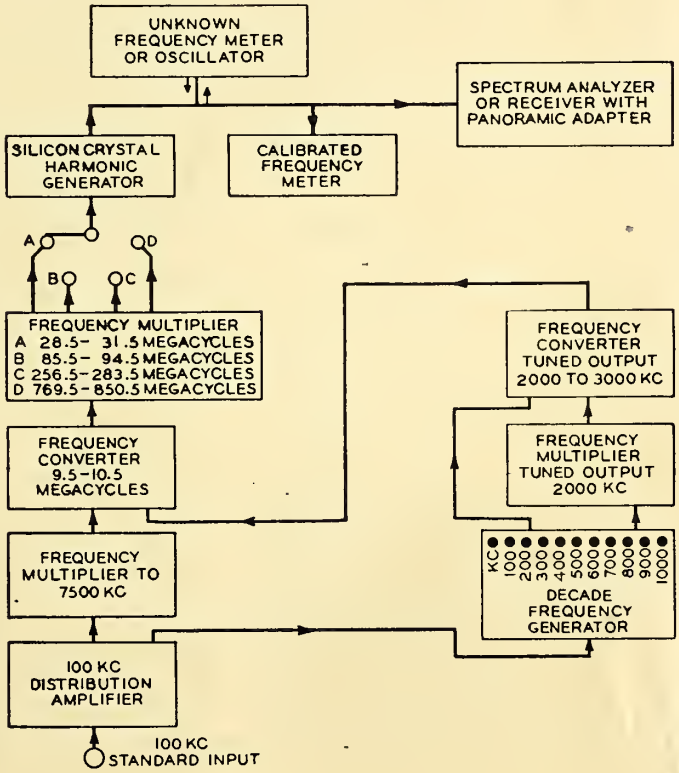

UNKNOWN FREQUENCY METER OR OSCILLATOR
SPECTRUM ANALYZER OR RECEIVER WITH PANORAMIC ADAPTER
SILICON CRYSTAL HARMONIC GENERATOR
CALIBRATED FREQUENCY METER
A
B
C
D
FREQUENCY MULTIPLIER
A 28.5- 31.5 MEGACYCLES
B 85.5- 94.5 MEGACYCLES
C 256.5-283.5 MEGACYCLES
D 769.5-850.5 MEGACYCLES
FREQUENCY CONVERTER TUNED OUTPUT 2000 TO 3000 KC
FREQUENCY CONVERTER 9.5-10.5 MEGACYCLES
FREQUENCY MULTIPLIER TUNED OUTPUT 2000 KC
FREQUENCY MULTIPLIER TO 7500 KC
KC 100 200 300 400 500 600 700 800 900 1000
DECADE FREQUENCY GENERATOR
100 KC DISTRIBUTION AMPLIFIER
100 KC STANDARD INPUT

Figure 3. Microwave frequency standard spot frequency coverage

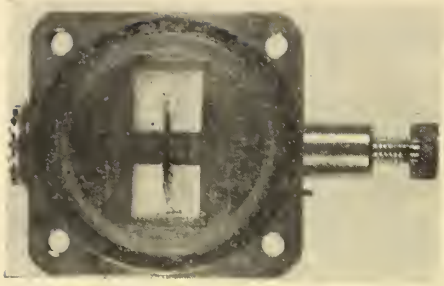


Figure 4. Crystal diode mounted in wave guide

prepared for every 100-kc step from 2.0 to 3.0 megacycles. The frequency to be measured is tuned in on the spectrum analyzer and an approximate value of the frequency is obtained from a calibrated frequency meter. The frequency standard then is set up for the proper frequency by reference to the tables. A pair of pips should appear on the screen of the receiver and by adjusting the power level of the standard or of the generator being measured they can be made equal in amplitude. At this point a check should be made to insure that the two frequencies are approximately equal and are not separated by twice the intermediate frequency. By bringing the frequencies into coincidence and increasing the dispersion it is possible to observe the beat notes on the screen. These beat notes appear as a filling-in of the response curve on the screen. It is possible with this method to determine the frequency of an oscillator to within one part in one million, which is considerably better than the stability of most microwave oscillators. If greater precision is warranted, it is possible to dispense entirely with the spectrum analyzer method and to measure the audio beat note of the two frequencies by any one of several methods.

Microwave Frequency Meters

Microwave frequency meters in common use are of three types

1. Coaxial line.
2. Cylindrical cavity.
3. Transition, which is a combination of 1 and 2.

Since these meters are resonant cavities the terms frequency meter, resonant cavity, and cavity often are used interchangeably. These meters may be of either the transmission type or the reaction type. The reaction-type meter is coupled into the main transmission line through a T section or through an iris in the side of the wave guide. The coupling is adjusted so the meter will present a high impedance in series with the wave guide when it is tuned to resonance, thereby decreasing the power transfer of the guide. The meter is adjusted to resonance by tuning for minimum output.

The transmission-type meter is inserted directly into the line and the energy must pass through the meter. Resonance in this meter is indicated by maximum power transfer. In Figure 9 are shown some typical microwave resonant-cavity frequency meters.

Accuracy of Calibration

Although the frequency from the microwave frequency standard is known to at least one part in 10 million, it is practically impossible to certify a frequency meter to this accuracy. Many factors enter into the exact determination and certification of the resonance frequency which limit the accuracy of the usual meter to about 1 part in 100 thousand or less.

Mechanical inaccuracies such as dial backlash, loose or faulty mode dampers, creepage of the metals or dielectrics, and lack of ruggedness are obvious limitations to the accuracy but are not the only factors which must be considered. When a precise determination of the resonance frequency is desired, such variables as temperature, humidity, and pressure may have to be considered. In addition to these factors the amount of external reactance which is coupled into the frequency meter will also affect the resonance frequency and must be taken into account or made negligible during the calibration. Finally, the sensitivity of the resonance-indicating device and the finite Q of the meter itself limit the accuracy to which resonance can be determined.

From rigorous field theory it is possible to develop equations which describe the behavior of a resonant cavity in terms of the electric and magnetic fields at chosen planes of reference. It is possible to write these equations in very simple form for the behavior of a cavity in the immediate vicinity of a resonance frequency. The equations given here are for the case of a cavity with electrically identical input and output couplings. The equations in this form are from the unpublished notes of David Kerns of the National Bureau of Standards. Similar expressions in different forms have been derived by other investigators.[1]

$$I_1 = I_2$$

$$V_1 = \frac{2Q_L}{\sqrt{T}}\left(\frac{1}{Q_0}+\frac{2j\Delta f}{f_0}\right)I_2 + V_2$$

I_1 and V_1 are, respectively, the linear measures of magnetic and electric fields at the input plane of reference. I_2 and V_2 are similar measures at the output plane. The physical meaning of the equations is given in essence by the fact that $V_1I_1^*$ is the input power; $V_2I_2^*$ is the output power from the cavity.

Q_0 is the unloaded Q of the cavity.
Q_L is the loaded Q measured with reflectionless loads at the planes of reference.
T is the ratio of the output power to the incident power at resonance and with reflectionless generator and load.
f_0 is the resonance frequency and
$f = f_0 + \Delta f$
I^* is the complex conjugate of I.

Any number of circuits can be imagined whose terminal voltages and currents are connected by these equations, the simplest being the complex impedance

$$\frac{2Q_L}{\sqrt{T}}\left(\frac{1}{Q_0}+\frac{2j\Delta f}{f_0}\right)$$

in series with a generator and a load. This, or any of the other circuit arrangements which can be derived from these equations, can be called the equivalent circuit of the cavity resonator in the immediate vicinity of a resonance frequency.

The resonance frequency of the transmission-type resonant cavity is the frequency at which maximum power transfer occurs. At this frequency the reactive component in the series impedance becomes zero. Δf in the basic equations becomes zero. The accuracy to which the cavity can be set to resonance is a function of the loaded Q of the cavity and the sensitivity of the indicating system. The higher the Q of the cavity and the smaller the percentage change in output power or voltage which can be observed in the indicator, the greater will be the accuracy of setting. The indicator used at the National Bureau of Standards for most frequency-meter calibrations is a spectrum-analyzer receiver. The height of the response on the oscilloscope screen is approximately proportional to the input power.

In the present notation a matched (reflectionless) impedance is represented by the normalized value unity. By solving the basic equations for a load impedance $V_2/I_2 = 1$, and a source impedance also equal to unity, one finds that the ratio of the power delivered to the load at a frequency f to that delivered at the resonance frequency f_0 is

$$\frac{P}{P_0} = \frac{1}{1 + Q_L^2\left(\frac{2\Delta f}{f_0}\right)^2}$$

where

$f = f_0 + \Delta f$

This ratio is also approximately equal to h/h_0, the ratio of the responses on the oscilloscope screen

$$\frac{\Delta f}{f_0} = \pm\frac{1}{2Q_L}\sqrt{\frac{h_0 - h}{h}} = \pm\frac{1}{2Q_L}\sqrt{\frac{\Delta h}{h}}$$

If $\Delta h/h$ is the smallest fractional change which can be detected in the indicator, then $\Delta f/f_0$ is the uncertainty in the resonance setting. With the indicating equipment now being used it is possible to detect changes in h of about 1 per cent.

For the typical case of a 10,000-megacycle transmission-type frequency meter with a loaded Q of approximately 10,000, the resonance frequency can be determined to within ± 0.05 megacycles.

Although this derivation and example is for a transmission-type meter, it has been found that when the same meter is used as a reaction-type instrument, the inaccuracy of setting is but slightly greater than that of the transmission type.

Figure 5. Silicon crystal diode

When greater accuracies are desired, some other method of indicating resonance must be used. It is possible to increase the accuracy by working on the steep sides of the resonance curve rather than at the relatively flat top. One means of accomplishing this is to inject a low frequency into the harmonic generating crystal along with the standard frequency. This low frequency is adjusted so it produces side-band markers on the microwave harmonic at approximately $f_0/2Q_L$ on each side of the carrier. It is possible to suppress the microwave carrier to a considerable extent by adjusting the level of the low-frequency input. If the response of the indicating system is uniform over the band width f_0/Q_L, resonance is indicated when these two markers are of equal amplitude. With this indicating system it is possible to set the frequency meter in the preceding example to within ± 0.02 megacycle at 10,000 megacycles per second.

An instrument capable of great accuracy in the setting of the resonance frequency is the cavity Q meter as developed by the staff of the MIT Radiation Laboratory.[2] By means of a frequency-modulated oscillator the resonance curve of the cavity is displayed on an oscilloscope screen. The same oscillator beating against the microwave harmonic from the frequency standard produces two bright spots on the resonance curve. By adjusting the intermediate-frequency amplifier of the Q meter these spots can be made to appear at the half-power points of the curve.

Cavity resonance is indicated when these spots are symmetrically placed on the curve. It is possible to set the meter to resonance to within a fractional error $\pm 0.01/Q_L$.

Frequency Pulling by External Reactance

During the calibration or use of any frequency meter it is essential for highest accuracy that the meter be isolated from external reactances by means of matched buffers. These buffers, usually resistive pads with an attenuation of about 10 decibels, are placed at both sides of the meter to prevent reactances in the line from "pulling" the resonance frequency of the meter. In calculating the extent of this pulling it is assumed that an impedance with a series reactance component jX (normalized value) appears at the output reference plane. For the moment it is assumed that the generator impedance is unity.

Because of this added reactance the reactive components in the circuit will now total zero at a frequency which is slightly different from the true resonance frequency. Again employing the basic cavity equations, the condition for resonance can be written

$$\frac{j4Q_L}{\sqrt{T}}\frac{\Delta f}{f_0} + jX = 0$$

or

$$\frac{\Delta f}{f_0} = -\frac{\sqrt{T}}{4Q_L}X$$

For applications of this equation to wave guide techniques it is convenient to express $\Delta f/f_0$ in terms of a voltage standing-wave ratio (VSWR), rather than in terms of the normalized reactance. From transmission-line theory it can be shown that the maximum magnitude of normalized reactance for a given VSWR on the line is $(r^2-1)/2r$ where r is the VSWR. Therefore

$$\frac{\Delta f}{f_0} = \pm\frac{\sqrt{T}}{8Q_L}\frac{r^2-1}{r}$$

This frequency shift is the maximum for a given VSWR existing at the output of the meter.

Any reactance at the generator side of the cavity will also cause a frequency pulling. The maximum possible frequency shift will be the sum of the two individual shifts.

Referring to the frequency meter of a previous example, $f_0 = 10{,}000$ megacycles, $Q_L = 10{,}000$, it will be assumed that the insertion loss of this meter is 6 decibels. A VSWR of 2.0 at the output side of the meter can produce a frequency shift of approximately ± 0.1 megacycle. If the meter also sees a similar VSWR at the input the possible frequency shift becomes ± 0.2 megacycle.

When a matched attenuator is placed between the meter and the rest of the circuit, the standing-wave ratio which the meter sees will be considerably reduced. For example, an infinite voltage standing-wave ratio will be reduced to 1.22 through a perfectly matched 10-decibel attenuator. If such attenuators are used with the meter in the previous example, the VSWR will be reduced from 2.0 to 1.07 and the frequency shift will be reduced to ± 0.02 megacycle.

The usual attenuator is not perfectly matched but does introduce standing waves, which may enhance the standing waves from other discontinuities. If the 10-decibel attenuator pads have a VSWR

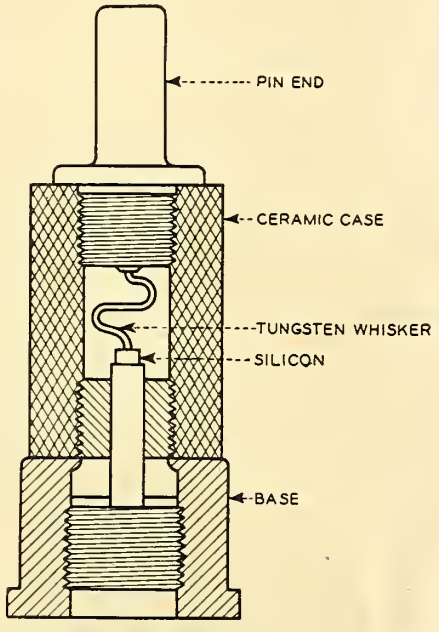


Figure 6. Schematic diagram of a crystal diode

of 1.10 or less, the maximum frequency shifts which can be expected will be

$$\pm 0.16 \frac{\sqrt{T}}{Q_L} f_0$$

(± 0.08 megacycle for the meter of the previous example).

It is apparent that in order to eliminate the possibility of frequency pulling, the meter should be isolated by resistive pads which are in themselves fairly well matched to the line, not only during the calibration but also during the use of the meter. If no suitable attenuators are available, the line should be adjusted so it appears flat looking either way from the frequency meter.

Effects of Temperature, Humidity, and Pressure

Changes of temperature, humidity, and pressure in general will affect the calibration of the cavity-type frequency meter. The actual percentage change in the resonance frequency for a given change in the foregoing factors will depend upon the type of construction and upon the materials in the meter.

The most obvious source of error lies in the change of cavity dimensions due to thermal expansion of the walls. From the principle of electrodynamic similitude[3] it follows that in the case of a perfectly conducting cavity, regardless of the shape of the cavity or the mode of oscillation, an increase in all the linear dimensions by a factor m will decrease the frequency by a factor $1/m$.

Neglecting second-order effects, this can be expressed for small changes in dimensions as

$$\frac{\Delta f}{f} = -\frac{\Delta L}{L} = -\alpha \Delta T$$

where $\Delta L/L$ is the fractional change in linear dimensions, α is the thermal coefficient of expansion for the wall material, and ΔT is the change in temperature. Since the percentage change in frequency for a given change in temperature is proportional to the thermal coefficient of expansion, a cavity made of invar will be much better than one of brass. The thermal coefficient of annealed invar ranges from 1×10^{-6} to 3×10^{-6} while the coefficient for brass is about 19×10^{-6}. If the temperature of the meter during calibration is constant to within ± 0.1 degrees centigrade and can be specified to within the same limits, the variations in resonance frequency due to thermal expansions will be about ± 0.02 megacycle at 10,000 megacycles per second for a brass cavity and ± 0.003 megacycle for an invar one.

It is possible to introduce compensating elements which will tend to compensate for changes in cavity dimensions so as to keep the resonance frequency constant over a certain range of temperature.

A change in the dielectric constant of the gas in the cavity also will shift the resonance frequency. Changes of temperature, pressure, and vapor pressure in the ambient atmosphere all may affect the dielectric constant of the gas. The extent of this influence will depend upon the type of construction and upon the gas in the cavity. In a sealed unit the type of gas and the moisture content of the gas are determined at the time of construction and remain constant as long as no moisture is condensed out and as long as no leaks develop.

The gas within an unsealed unit is approximately the same as the ambient atmosphere and can be made, for all practical purposes, identical to this atmosphere through a thorough flushing with the ambient air.

A fractional change $\Delta\epsilon/\epsilon$ in the dielectric constant will produce a fractional change in frequency of approximately $\Delta f/f = -1/2(\Delta\epsilon/\epsilon)$. The dielectric constant of air for normal ranges of temperature and pressure is given to a close approximation by

$$\epsilon = 1 + \frac{208}{T}\left(P + \frac{4800}{T} Pw\right) 10^{-6}$$

where P and Pw are respectively the total pressure and the water vapor pressure in millimeters of mercury and T is the temperature of the air in degrees Kelvin.

If the cavity has been filled and sealed with completely dry air or a dry nonpolar gas, Pw will be zero and there will be no change in dielectric constant with respect to temperature. However, if the cavity had been filled and sealed with air containing some water, the change in dielectric constant would depend upon the amount of water vapor present. For temperatures above the dew point of the atmosphere in the cavity, this change is small. The fractional change in frequency for small changes in temperature is approximately

$$\frac{\Delta f}{f} = 1/2 \frac{Pw}{T^3} \Delta T$$

If the temperature falls below the dew point, moisture will collect on the surfaces in the cavity and the frequency error may become quite large.

Changes in external pressure have no effect on the resonance frequency of a sealed cavity unless the pressure difference becomes large enough to distort the cavity.

Unsealed cavities cannot be calibrated accurately because the temperature and humidity of the air in the cavity cannot be specified with great precision. Not only is the accuracy of the unit limited in use because of temperature and humidity variations, but in addition, the initial calibration against a primary standard is limited in accuracy by the same factors. Changes in atmospheric pressure also affect the resonance frequency of an unsealed cavity. The pressure in the cavity however, is identical to the ambient pressure which can be measured with good precision and corrected for, if necessary.

As an example of the magnitude of these variations it will be assumed that the operating conditions for an unsealed invar cavity are 25 degrees centigrade, 50 per cent relative humidity, and 760 millimeters of mercury. A temperature increase of 1 degree centigrade will cause a fractional change in frequency of -3×10^{-6} through thermal expansion of the wall and $+2\times10^{-7}$ through changes in the dielectric constant of the air. An increase of 2 per cent in the value of the relative humidity will cause a fractional frequency change of -3×10^{-6}. A change of 1 centimeter of mercury in the pressure will introduce a fractional change of -4×10^{-6}.

In order for a precision meter to be

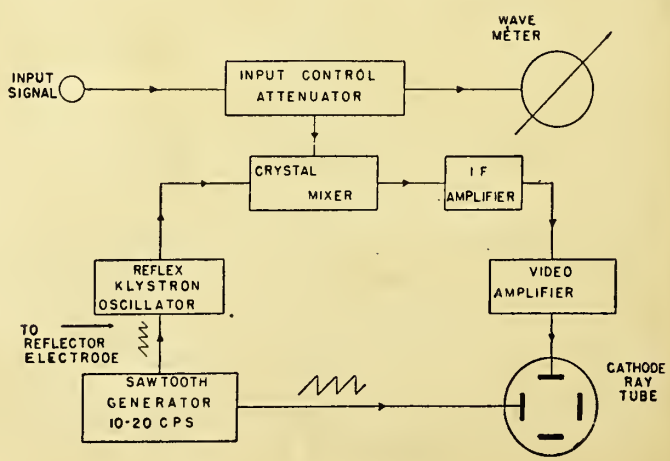
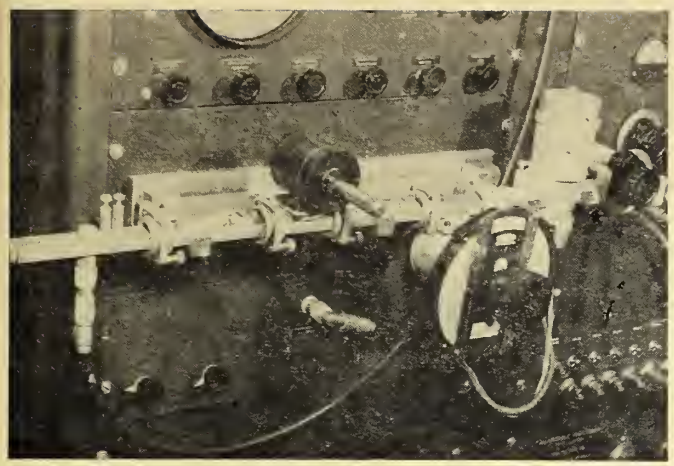


Figure 7. Block diagram of spectrum analyzer



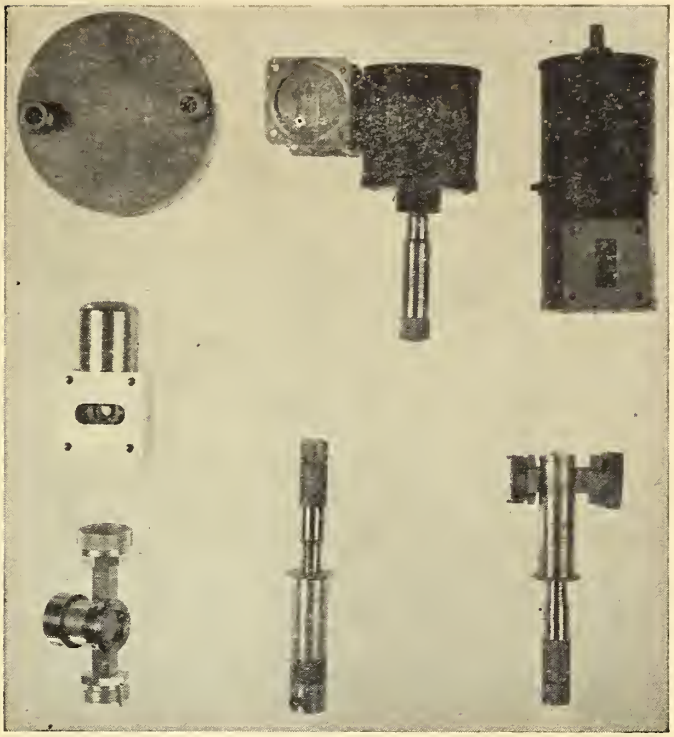


Figure 8 (above). Spectrum analyzer with interchangeable wave-guide assemblies

Figure 9. Microwave frequency meters

relatively free of external effects the design of such a meter suitable for use as a secondary standard should embody the following features:

1. The meter should be rigidly constructed of materials which will not creep or change dimensions.

2. The materials should have a low coefficient of thermal expansion and should exhibit no anomalies in the coefficient of expansion with temperature cycling.

3. Temperature compensation may be employed.

4. The electrical design should be consistent with high Q requirements and there should be no interfering modes of oscillation. Mode suppressors should be electrically and mechanically stable.

5. All moisture should be removed from the cavity including that absorbed in the walls and in the damping material. The cavity should be filled with a completely dry inert gas and sealed.

Conclusion

In conclusion, it may be of interest to consider some possible future developments and applications in the field of microwave frequency measurements. Some of the developments to be mentioned are already well underway.

There is an urgent need for improved cavities having very great stability. One method of attack on this problem is the consideration of cavities made of quartz in order to reduce temperature variations. The quartz is silver plated in order to form a metallic wall for the cavity. A solid quartz cavity, of course, will be impervious to moisture.

Cavities, particularly highly stable ones, can be used to stabilize klystron oscillators by means of a microwave discriminator circuit.[4] This results in a standard-frequency generator having considerable power output. In this way, the cavity is used as an active rather than a passive frequency standard. Such an oscillator can be quite useful in frequency measurements as an interpolation oscillator just as such a unit is used in low-frequency technique. It also has been proposed by the MIT Radiation Laboratory to use such cavity-stabilized oscillators for microwave communication systems in which an enormous number of radio channels are made available in a given range of the frequency spectrum because of the great stability achieved. It would then seem to be possible to use a cavity-stabilized transmitter in much the same way crystal-controlled transmitters are used at lower frequency, except that turning over all the channels is achieved with only one cavity.

By means of resonant cavity techniques and accurate frequency measurement methods, a general method for the measurement of complex dielectric constants is obtained.[5] When a dielectric substance is inserted in a resonant cavity, measurement of the change in resonance frequency and the change in Q or band width of the cavity gives one a measurement of the dielectric constant and the power factor of the substance introduced into cavity. This method gives an extremely interesting way of measuring relative humidity of atmospheric air and of the index of refraction of atmospheric air as needed for the study of microwave propagation. One method of making a microwave refractometer using these principles is to use two cavity-stabilized oscillators which are tuned to zero beat. Introduction of atmospheric air in one of the cavities then changes the frequency of the oscillator and a measurement of the beat note gives the index of refraction directly.

Another application of resonant-cavity frequency measurements is to a new and precise determination of the velocity of radio waves, or determination of the universal constant c by measuring the resonance frequency and dimensions of a cavity with extreme precision. This investigation is being carried out at Stanford University under Professor W. W. Hansen. Here even the effect of finite conductivity of the walls of the cavity must be corrected for by adding the skin depth of penetration of the current on the walls of the cavity to the dimensions of the cavity.

It is becoming more and more urgent to extend the range of the Bureau's microwave frequency standard above 30,000 megacycles per second. This problem is being attacked by development of a lighthouse-tube frequency multiplier circuit which will provide, through the preceding multiplier chain, crystal-controlled output in the 2,400-megacycle range. By development of suitable crystal rectifier mounts, harmonic generation at a useful power level should be possible to 75,000 megacycles or higher. It is also possible to use Sperry klystron frequency-multiplier tubes to obtain crystal-controlled power at 10,000 megacycles at a level of approximately one milliwatt. By harmonic generation it then should be possible to obtain useful outputs to 100,000

megacycles and higher. At these extreme high frequencies it probably will be desirable to introduce new frequency measuring instruments along the lines of optical equipment. In this connection, microwave interferometer and echelon gratings have been built and have become practical.

In the higher frequency ranges, many substances have spectroscopic absorption lines.[6] A familiar example is ammonia gas which has a very strong absorption line at about 24,000 megacycles. This line is so broad due to molecular collisions that, at atmospheric pressure, the absorption is quite strong even at 9,000 megacycles. Many other gases have been shown to exhibit such absorption and it is likely that additional gases will be found at still higher frequencies, in what can be called the millimeter wave-length bands. Under the proper conditions of low pressure some spectrum lines are extremely narrow so that it may be possible to use such lines as frequency standards and even to control the frequency of oscillators or to use the lines in conjunction with cavities. The use of spectrum lines as microwave frequency standards is, in many ways, analogous to the use of optical spectrum lines as length standards by relating their wavelength to the standard meter. A standard-frequency generator like the one described in this paper can be used to measure accurately the frequencies of absorption lines. However, a quite compact standard-frequency generator, for providing precision marker frequencies to be used for microwave spectroscopy measurements, can be constructed by means of a quartz crystal oscillator and frequency multipliers driving Sperry klystron multiplier tubes.

At the present time, it is hoped that microwave frequency standards at institutions other than the National Bureau of Standards can be standardized through utilization of the standard frequency emissions from station *WWV*. This is particularly easy if the microwave frequency standard is of a type similar to that used at the National Bureau of Standards, since it will only be necessary to adjust the quartz crystal oscillator or other oscillator, serving as the starting point of the frequency multiplying chain, to the frequency of *WWV* by means of the proper receiver and intercomparison circuits. In the future, if suitable absorption lines can be found for use as frequency standards, it may be possible to calibrate the various types of microwave frequency standards, wherever located, through the use of spectroscopic frequency standards. At the present time frequency measurements and calibrations of frequency meters or voltage sources can be obtained from the National Bureau of Standards as a calibration service similar to the standards services rendered by the Bureau in other fields of measurement.

References

1. Microwave Electronics, **J. C. Slater.** *Reviews of Modern Physics* (New York, N. Y.), volume 18, 1946, page 441.

2. Microwave Measurements and Test Equipments, **F. J. Gaffney.** *Proceedings*, Institute of Radio Engineers (New York, N. Y.), volume 34, 1946, page 775.

3. Electromagnetic Theory (book), **J. A. Stratton.** McGraw-Hill Book Company, New York, N. Y., 1941.

4. Electronic Frequency Stabilization of Microwave Oscillators, **R. V. Pound.** *Review of Scientific Instruments* (New York, N. Y.), volume 17, 1946, page 490.

5. Resonance Methods of Dielectric Measurements at Centimetre Wavelengths, **F. Horner.** *Journal*, Institution of Electrical Engineers (London, England), volume 93, 1946, part 3, page 53.

6. The Absorption of Microwaves by Gases, **W. D. Hershberger.** *Journal of Applied Physics* (New York, N. Y.), volume 17, 1946, pages 495, 814.

Preprinted from
TRANSACTIONS of the American
Institute of Electrical Engineers
33 West 39th St., New York, N. Y

Statements and opinions given in papers and discussions appearing in AIEE TRANSACTIONS are the expressions of contributors, for which the Institute assumes no responsibility.

Printed in the United States of America

6/25/'48 Volume 67, 1948